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Micromechanical model of nacre tested in tension

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A modified shear lag theory is used to model the tensile behavior of *Pinctada* nacre. A two-dimensional model is used to analyze the stress transfer between the aragonite platelets of nacre assuming that the ends of the platelet are not bonded with the organic matrix. Elastic-perfectly plastic behavior of the organic matrix is assumed. A model for stress transfer between the platelets when the matrix between the platelets starts behaving plastically is developed. It is assumed that nacre fails when the matrix breaks after the ultimate shear strain in the matrix is exceeded. This theory can be used to model the stress transfer in platelet reinforced composites at high volume fractions.

1. Introduction

Nacre is a molluscan shell material containing 95% calcium carbonate by volume that is held together by an organic matrix. The calcium carbonate phase is in the form of polygonal-shaped aragonite platelets [1]. The platelets are separated on the sides and the ends by a thin layer of a protein-polysaccharide matrix [1, 2]. The arrangement and the aspect ratio of the platelets are different in different mollusc species.

The platelets in nacre are about 0.5 μm thick with lengths of about 4 μm. The platelets in *Pinctada* nacre are arranged in the form of lamellae with neighboring platelets overlapping by 50% [1, 3]. This microarchitecture leads to a composite with toughness and strengths an order of magnitude higher than that of clumped aragonite mineral or that of the organic alone. In order to mimic these properties of nacre, we need to have a fundamental understanding as to the origin of the mechanical behavior in nacre. The elastic modulus of nacre has been modeled earlier assuming both overlapped platelets and non-overlapped platelets [3]. However, the stress transfer between the overlapped platelets as well as the plastic behavior of nacre tested in tension has not been investigated.

Therefore, a simple 2-D shear lag model is used to investigate the stress transfer between the overlapped platelets. Since the shape and organization of the mineral as well as the mechanical properties of *Pinctada* nacre and its constituents are known, a model for the tensile behavior of *Pinctada* nacre is developed.

2. Analytical model

The stress transfer in a unit cell consisting of a quarter of two overlapped platelets in adjacent layers is modeled (Fig. 1a). The space between the sides of the platelets occupied by the matrix is exaggerated to make the choice of co-ordinates clear. The organic matrix at the sides of the two overlapped platelets is assumed to be in the form of shear springs.

The following assumptions are made in the analysis:

1) the platelets are assumed to be of uniform width, rectangular in shape and isotropic
2) the platelets are uniformly arranged such that they are fully overlapped
3) the platelets carry the axial stresses and the stress is transferred from one platelet to the other by shear in the matrix
4) the matrix at the ends of the platelets contributes little to the stress transfer

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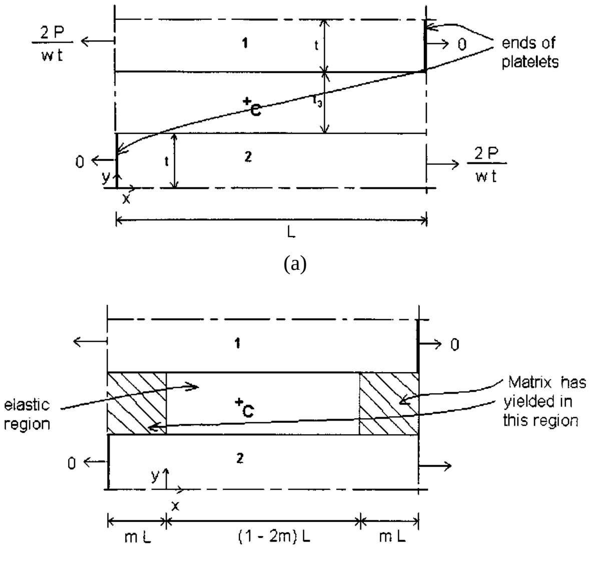

Figure 1 (a) The unit cell made up of two overlapped bone mineral platelets and (b) The unit cell made up of two overlapped bone mineral platelets divided into three regions. At the two ends of the unit cell, the matrix is deforming plastically.

5) shear stresses in the organic on the sides of the platelets is assumed to be constant

6) deformation and stresses perpendicular to the direction of applied stresses are ignored

7) elastic modulus is derived assuming that no interface is present and bonding is perfect

8) residual stress effects are neglected

9) the organic is assumed to be elastic-perfectly plastic in shear and fails when ultimate shear strain is reached

The platelets have a thickness '$2t$', length '$2L$' and width 'w'. The separation between the platelets on the sides is 't_3'. Because of symmetry, the stress transfer in two quadrants of the two overlapped platelets is modeled (Fig. 1a). The subscripts '1', '2' and '3' denote the properties of platelet '1', platelet '2' and the matrix '3' respectively.

The equilibrium between the axial stress, σ_x, and the shear stress τ_{xy} $(=\tau)$, is

$$\frac{\partial \sigma_x}{\partial x} = -\frac{\partial \tau}{\partial y} \tag{1}$$

Integrating Equation 1 over the thickness of the platelets in the 'y' direction and dividing by the thickness we get:

$$\frac{d\sigma_{1x}}{\text{dx}} = -\frac{\tau_{\text{ix}}}{t} \quad \text{and} \quad \frac{d\sigma_{2x}}{\text{dx}} = \frac{\tau_{\text{ix}}}{t} \tag{2}$$

where σ_{1x} and σ_{2x} represent the average value of axial stress over the thickness of the platelets '1' and '2' respectively in the 'x' direction. 'τ_{ix}' is the interfacial shear stress (shear stress on the surfaces of the platelets) in the 'x' direction. The axial stresses in the platelets '1' and '2', σ_{1x} and σ_{2x}, are assumed to remain constant in the 'y' direction. One platelet is gaining load and the other platelet is giving off load resulting in the negative sign in front of one equation. This is the same as the stress transfer equation that was derived for platelets earlier [4].

The difference between the displacements of the platelets '1' and '2' gives the shear displacement in the matrix assuming that the stress and displacement in the 'y' direction is negligible. This results in:

$$u_{2x} - u_{1x} = t_3\gamma_{3x} \tag{3}$$

where 'u_{1x}' and 'u_{2x}' are the displacements of platelet '1' and '2' respectively in the 'x' direction and 'γ_{3x}' is the 'x' component of the shear strain in the matrix between the two platelets.

Differentiating Equation 3 with respect to 'x', substituting for the shear strain in terms of the shear modulus and shear stress (τ_{ix}) and using hooke's law ($\varepsilon_{1x} = \sigma_{1x}/E_1$ and $\varepsilon_{2x} = \sigma_{2x}/E_2$) we have,

$$\frac{\sigma_{2x}}{E_2} - \frac{\sigma_{1x}}{E_1} = -\frac{t\, t_3}{G_3}\frac{d^2\sigma_{1x}}{dx^2} \tag{4}$$

Since we assume that the load carried by the two platelets is '$2P$', we have,

$$\sigma_{1x} + \sigma_{2x} = \frac{2P}{\text{wt}} \tag{5}$$

substituting Equation 5 in Equation 4,

$$\frac{d^2\sigma_{1x}}{\text{dx}^2} - \frac{G_3}{t\, t_3}\left(\frac{1}{E_1} + \frac{1}{E_2}\right)\sigma_{1x} = -\frac{2P}{\text{wt}E_2}\frac{G_3}{t\, t_3} \tag{6}$$

The solution of the differential Equation 6, assuming both platelets have the same elastic modulus ($E_1 = E_2$), is of the form

$$\sigma_{1x} = \frac{P}{\text{wt}} + A\cosh\alpha x + B\sinh\alpha x \tag{7}$$

where 'A' and 'B' are constants to be determined from the boundary conditions and

$$\alpha = \sqrt{\frac{G_3}{t_3\, t}\frac{2}{E_1}}$$

Equation 7 gives the variation in the axial stress in platelet '1' along the 'x' axis.
The following boundary conditions are used to obtain the constants A and B: -

$$\sigma_{1x}(\text{at}\, x = L) = \sigma_{2x}(\text{at}\, x = 0) = 0$$

$$\sigma_{1x}(\text{at}\, x = 0) = \sigma_{2x}(\text{at}\, x = L) = 2P/\text{wt}$$

Using the boundary conditions given above we have

$$A = \frac{P}{\text{wt}}; \quad B = -\frac{P}{\text{wt}}\left(\frac{1 + \cosh(\alpha L)}{\sinh(\alpha L)}\right); \tag{8}$$

Differentiating Equation 7 and using Equation 2 one can obtain the shear stresses on the platelet surface. This gives:

$$\tau_{ix} = -t\alpha(A\sinh(\alpha x) + B\cosh(\alpha x)) \quad (9)$$

Using Equation 5 and Equation 7, the axial stresses in platelet '2' can be determined. The average stresses in the real platelets are obtained by averaging the stresses from Equation 7 over the length of the real platelet.

The average stresses carried by the real platelet '1' and '2' in the overlap and non- overlap regions are: -

$$\overline{\sigma_{1x}} = \frac{1}{L}\int_0^L \sigma_{1x}\ \mathrm{dx} = \frac{P}{\mathrm{wt}}$$
$$(10)$$
$$\overline{\sigma_{2x}} = \frac{1}{L}\int_0^L \sigma_{2x}\ \mathrm{dx} = \frac{P}{\mathrm{wt}}$$

where the bar on top of the stresses represents the average stresses.

The total strain in the composite is obtained in terms of 'P/wt' by using the one-dimensional Hooke's law ($\varepsilon = \sigma/E$). The total strain is given by:

$$\varepsilon_{\mathrm{total}} = \frac{\dfrac{L\overline{\sigma_{1x}}}{E_1} + \dfrac{\tau_i|_{x=0}t_3}{G_3}}{L}$$

or

$$\varepsilon_{\mathrm{total}} = \frac{\left(\dfrac{L}{E_1} + \dfrac{t\,t_3\alpha}{G_3}\left(\dfrac{1+\cosh(\alpha L)}{\sinh(\alpha L)}\right)\right)\dfrac{P}{\mathrm{wt}}}{L} \quad (11)$$

Neglecting the axial stress carried by the organic, the elastic modulus of the composite is,

$$E_c = \frac{V_{1x}\,\overline{\sigma_{1x}} + V_{2x}\,\overline{\sigma_{2x}}}{\varepsilon_{\mathrm{total}}} = \frac{2tL}{(2t+t_3)\left(\dfrac{L}{E_1} + \dfrac{t\,t_3\alpha}{G_3}\left(\dfrac{1+\cosh(\alpha L)}{\sinh(\alpha L)}\right)\right)} \quad (12)$$

where V_{1x} and V_{2x} are the volume fractions of the platelets '1' and '2' respectively. This equation is valid only at high volume fractions of platelets (>90%), as is the case in nacre.

The load is transferred elastically till the shear stress in the matrix at the ends of the platelets reaches the shear yield stress (τ_{my}). Using Equation 9, with $\tau_{ix} = \tau_{my}$ (at $x = 0$), we can obtain the applied stress 'P/wt' when the matrix at the ends of the unit cell begins to yield.

$$\frac{P}{\mathrm{wt}} = \frac{\tau_{my}}{t\alpha}\frac{\sinh(\alpha L)}{(1+\cosh(\alpha L))} \quad (13)$$

Using this value of 'P/wt' in Equation 11, we can obtain the strain at which the organic starts to yield.

When the shear stress at the ends of the platelets '1' or '2' reaches the yield shear stress of the matrix, the matrix is assumed to strain in shear at a constant stress, which is the shear yield stress (τ_{my}) of the matrix. The unit cell is divided into three regions based on the stress transfer that takes place. The region at the two ends of the unit cell, of length 'mL', is the region where the shear stress has reached the shear yield stress of the matrix (τ_{my}). 'm' is the fraction of the overlapped length 'L'. The value of 'm' varies between 0 and 0.5. When the shear stress over the surface of the entire platelet has reached the yield shear stress of the matrix, $m = 0.5$. The region of length '$(1-2m)\,L$' at the center of the unit cell, where the elastic stress transfer takes place has its origin at 'x' (Fig. 1b).

The total stress transferred between the platelets is given by:

$$\sigma_{1x} + \sigma_{2x} = \frac{2P}{\mathrm{wt}} + \frac{2\tau_{my}\mathrm{mL}}{t} \quad (14)$$

After the matrix has yielded, Equation 14 is used instead of Equation 5 in the derivation for the axial stress transfer in the region where the stress transfer is elastic. Using the same steps as previously and assuming $E_1 = E_2$, we get,

$$\sigma_{1x} = \frac{P}{\mathrm{wt}} + \frac{\tau_{my}\mathrm{mL}}{t} + A\cosh\alpha x + B\sinh\alpha x \quad (15)$$

where 'A' and 'B' are constants to be determined from the boundary conditions and

$$\alpha = \sqrt{\frac{G_3}{t_3\,t}\frac{2}{E_1}}$$

In the elastic region at the center of the unit cell (Fig. 1b), Equation 15 is used for the axial stress transfer along platelet '1'. The boundary conditions used are changed to:

$$\sigma_{1x}(\text{at } x = (1-2m)L) = \sigma_{2x}(\text{at } x = 0) = \tau_{my}\mathrm{mL}/t$$
$$\sigma_{1x}(\text{at } x = 0) = \sigma_{2x}(\text{at } x = (1-2m)L) = 2P/\mathrm{wt} + \tau_{my}\mathrm{mL}/t$$

Using the boundary conditions given above we have

$$A = \frac{P}{\mathrm{wt}}; \qquad B = -\frac{P}{\mathrm{wt}}\left(\frac{1+\cosh(\alpha(1-2m)L)}{\sinh(\alpha(1-2m)L)}\right); \quad (16)$$

Differentiating Equation 15 and using Equation 2 one can obtain the shear stresses on the platelet surface. This gives:

$$\tau_{ix} = -t\alpha(A\sinh(\alpha x) + B\cosh(\alpha x)) \quad (17)$$

The shear stress at '$x = 0$' or at '$x = (1-2m)L$' is the shear yield stress of the matrix 'τ_{my}'. Using this,we find the stress 'P/wt' that is transferred in the elastic region. This gives,

$$\frac{P}{\mathrm{wt}} = \frac{\tau_{\mathrm{my}}}{t\alpha}\frac{\sinh(\alpha(1-2m)L)}{(1+\cosh(\alpha(1-2m)L))} \tag{18}$$

It is seen that as the value of 'm' tends to 0.5, 'P/wt' goes to zero.

The stress in the platelet '2' is obtained from Equation 14. The average stress in the platelets are obtained by integrating between '$x = 0$' and '$x = (1-2m)\ L$' and dividing by the length of the region where elastic stress transfer takes place. This gives: -

$$\overline{\sigma_{1x}} = \frac{P}{\mathrm{wt}} + \frac{\tau_{\mathrm{my}}\mathrm{mL}}{t}$$

$$\overline{\sigma_{2x}} = \frac{P}{\mathrm{wt}} + \frac{\tau_{\mathrm{my}}\mathrm{mL}}{t} \tag{19}$$

In the region at the two ends of the unit cell where the yield stress is constant,

$$\frac{d\sigma_{1x}}{\mathrm{dx}} = -\frac{\tau_{\mathrm{my}}}{t} \quad \text{and} \quad \frac{d\sigma_{2x}}{\mathrm{dx}} = \frac{\tau_{\mathrm{my}}}{t} \tag{20}$$

Integrating these with respect to 'x', and using the boundary conditions, (i) at $x = 0$; $\sigma_{1x} = 2P/\mathrm{wt} + \tau_{\mathrm{my}}\mathrm{mL}/t$ and (ii) at $x = 0$; $\sigma_{2x} = \tau_{\mathrm{my}}\mathrm{mL}/t$, we get,

$$\sigma_{1x} = \frac{2P}{\mathrm{wt}} + \frac{\tau_{\mathrm{my}}(\mathrm{mL}-x)}{t} \quad \text{and}$$

$$\sigma_{2x} = \frac{\tau_{\mathrm{my}}(\mathrm{mL}+x)}{t}$$

The average stress in the platelets '1' and '2' in this region is obtained by integrating the stresses from '$x = -\mathrm{mL}$' to '$x = 0$'. They are given as:

$$\overline{\sigma_{1x}}\,(\text{shear yield region}) = \frac{2P}{\mathrm{wt}} + \frac{3\tau_{\mathrm{my}}\mathrm{mL}}{2t}$$

$$\overline{\sigma_{2x}}\,(\text{shear yield region}) = \frac{\tau_{\mathrm{my}}\mathrm{mL}}{2t} \tag{21}$$

The shear strain in the matrix in the region where the matrix is yielding at a constant shear stress (τ_{my}) is calculated from the effective shear modulus (G^1_{3x}). The effective shear modulus is defined as the slope of the line connecting the shear stress—shear strain curve after the matrix has yielded to the origin G^1_3 (Fig. 2). It is derived as follows:

$$\gamma_{3x} = \frac{\tau_{\mathrm{my}}}{G^1_{3x}} \tag{22}$$

Differentiating Equation 22, with respect to 'x', we get,

$$\frac{d\gamma_{3x}}{\mathrm{dx}} = \tau_{\mathrm{my}}\frac{d}{\mathrm{dx}}\left(\frac{1}{G^1_{3x}}\right) \tag{23}$$

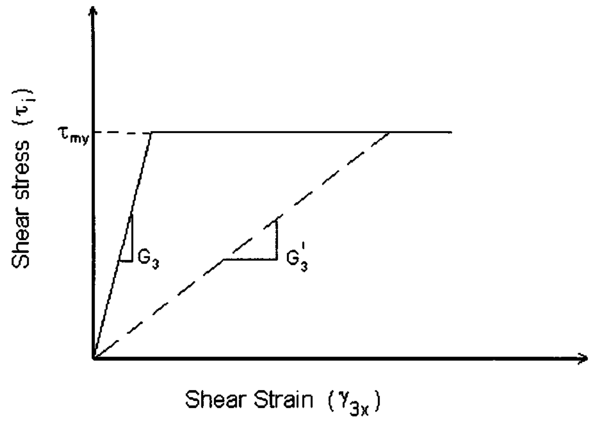

Figure 2 Idealized shear behavior of the organic. The shear modulus before (G_3) and the effective shear modulus after (G^1_3) the matrix yields.

Using Equation 23 in Equation 4 and using Hooke's law ($\varepsilon = \sigma/E$), we obtain,

$$\frac{\sigma_{2x}}{E_2} - \frac{\sigma_{1x}}{E_1} = t_3\tau_{\mathrm{my}}\frac{d}{\mathrm{dx}}\left(\frac{1}{G^1_{3x}}\right)$$

Substituting Equation 14 into the above equation,

$$t_3\tau_{\mathrm{my}}\frac{d}{\mathrm{dx}}\left(\frac{1}{G^1_{3x}}\right) = \frac{2P}{\mathrm{wt}E_1} + \frac{2\tau_{\mathrm{my}}\mathrm{mL}}{tE_1} - \frac{2\sigma_{1x}}{E_1} \tag{24}$$

Differentiating Equation 24 with respect to 'x' then substituting Equation 20, we obtain,

$$\frac{d^2}{\mathrm{dx}^2}\left(\frac{1}{G^1_{3x}}\right) = \frac{2}{t\,t_3E_1}$$

Integrating the above equation twice with respect to 'x', we get,

$$\frac{1}{G^1_{3x}} = \frac{x^2}{t\,t_3E_1} + K_1\mathrm{x} + K_2 \tag{25}$$

Using the following two boundary conditions in the above equation:

$$\text{At } x = 0,\ G^1_{3x} = G_3$$

$$\text{At } x = 0,\ \sigma_{1x} = 2P/\mathrm{wt} + \tau_{\mathrm{my}}\mathrm{mL}/t \ \ \text{(in Equation 24)}$$

We obtain,

$$\frac{1}{G^1_{3x}} = \frac{x^2}{t\,t_3E_1} - \frac{2(P/\mathrm{wt})x}{t_3\,E_1\,\tau_{\mathrm{my}}} + \frac{1}{G_3} \tag{26}$$

The total strain in the composite is given by:

$$\varepsilon_{\text{total}} = \frac{\left[\begin{array}{c}\dfrac{\text{mL}\,\overline{\sigma_{1x}}\text{ (shear yield region)}}{E_1} + \dfrac{(1-2m)L\,\overline{\sigma_{1x}}\text{ (elastic region)}}{E_1} \\ + \dfrac{\text{mL}\,\overline{\sigma_{2x}}\text{ (shear yield region)}}{E_1} + \tau_{\text{my}}\, t_3 \dfrac{1}{G^1_{3x}\big|_{x=-(\text{mL})}}\end{array}\right]}{L}$$

The above can be simplified to,

$$\varepsilon_{\text{total}} = \frac{\left[\dfrac{L}{E_1}\left(\dfrac{P}{\text{wt}} + \dfrac{\tau_{\text{my}}\text{mL}}{t}\right) + \tau_{\text{my}}\, t_3 \dfrac{1}{G^1_{3x}\big|_{x=-(\text{mL})}}\right]}{L} \tag{27}$$

It is seen that for a particular value of 'm', we can obtain 'P/wt' from Equation 18 and hence we can calculate $\varepsilon_{\text{total}}$.

The total stress σ_c is obtained by balancing the applied load at any cross-section in the composite. It is given by:

$$\sigma_c = V_{1x}\,\overline{\sigma_{1x}} + V_{2x}\,\overline{\sigma_{2x}} = \left(\frac{\left(\dfrac{P}{\text{wt}} + \dfrac{\tau_{\text{my}}\text{mL}}{t}\right)2t}{2t + t_3}\right) \tag{28}$$

The applied stress 'σ_c' is determined for a particular 'm' after using the value for 'P/wt' from Equation 18.

The composite is assumed to fail when the ultimate shear strain of the matrix is exceeded.

3. Results and discussion

A simple shear lag theory has been developed that takes into account the interactions of the stresses between the overlapped platelets. In this study, a closed form solution has been obtained to relate the elastic modulus of the composite to the material properties. The shear lag theory developed here is applicable only to high volume fractions of platelet reinforcements (>90%). This is because we have not considered the stress transferred in the overlapped portion of the unit cell (Fig. 1). The previous shear lag theories that have been developed [5, 6] only accounted for the interactions between overlapped fibers and not between overlapped platelets. None of the theories developed earlier has attempted to develop closed form solutions for the elastic moduli as high volume fractions (>90%) are not possible with fiber reinforcements. We have analyzed the stress transfer when the shear yield stress remains constant. Previous studies have analyzed the effect of work hardening of the matrix and debonding after the shear yield stress is reached [5].

The volume fraction of the platelets is 0.95 while the average aspect ratio of the polygonal platelets is assumed to be 8 along the plane of the platelets. The thickness as well as the length of the aragnonite platelets varies between the lamellar layers and only the average values are used here (0.25 μ for half the thickness and 2 μ for half the length as shown in the unit cell—Fig. 1a). The aragonite platelets are orthorhombic with varying elastic moduli along the different crystal planes [7]. The c-axis of the aragonite platelets in nacre is perpendicular to the plane of the layers, while the a-axis is oriented in different directions in the plane of the layers [8]. The elastic modulus of a synthetic aragonite platelet in the plane of the nacreous layers varies between 77 and 148 GPa [7]. The elastic moduli of the aragonite platelets in nacre are most likely lower than those determined for a synthetic crystal of aragonite. The average value of the elastic modulus of the aragonite platelets along the plane of the platelets, E_1, is assumed to be 100 GPa [3]. We assume that the ultimate strength of the aragonite platelets is not reached during tensile testing. The thickness of the organic matrix based on its volume fraction of 0.05% is calculated to be 0.028 μ. The shear moduli of the wet and dry matrix are experimentally found to be 1.4 and 4.6 GPa respectively [3]. The interfacial shear stresses at which the wet and dry matrix yields (τ_{my}) are experimentally determined to be 37 and 46 MPa respectively [3]. The mechanical behavior of nacre in shear has been assumed to be elastic-perfectly plastic (Fig. 2). This is a simplification as the individual organic matrix fibers in nacre are made of domains or loops that are pulled open at different strains [9]. However, the average shear behavior of a group of matrix fibers can be represented by the idealized elastic-perfectly plastic model (Fig. 2).

The elastic modulus predicted by the simple model (Equation 12) for the wet and dry nacre is 63 and 75 GPa respectively. This compares with the experimental values of 64 and 73 GPa for the wet and dry nacre in the across direction [3]. It is noted that the theoretical values are within 3% of the experimental values. The elastic modulus of nacre is sensitive to the shear modulus and thickness of the organic matrix, the aspect ratio and the elastic modulus of the aragonite platelets (Equation 12). Discrepancies of upto 10% in each of these parameters will cause a maximum difference of 15% in the calculated elastic modulus values. We have assumed no stress transfer through the ends of the platelet in the simple model. Based on the apparent agreement between the theoretically predicted values and the experimental measurements for the elastic modulus, the contribution of the matrix at the ends of the platelets would seem to be negligible. This would indicate that the elastic modulus of the organic matrix at the ends of the platelets is lower than that on the sides of the platelets. The effect of an interphase with

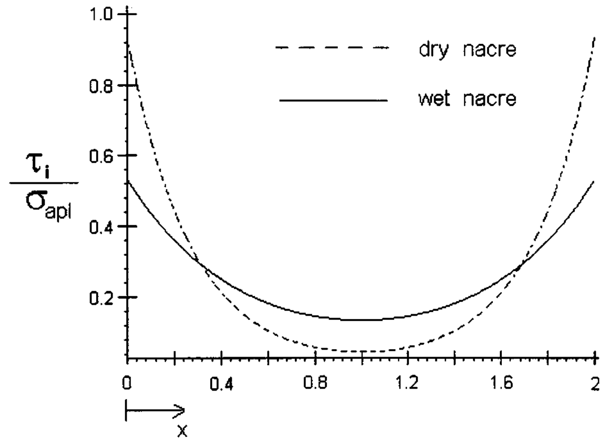

Figure 3 Shear stress transfer along the surface of platelet '1' from the right end to the left end of the unit cell.

mechanical properties lower than that of the bulk organic matrix as well as the effects of bonding between the platelets and the matrix has not been considered. A matrix fiber with low volume fraction that acts as a shear spring made of several domains or loops cannot cover the entire platelet surface. This would mean that stress transfer would occur at discrete points on the platelet surface and not over the entire surface resulting in a decrease in the elastic modulus. This has not been considered in this study.

The shear stresses along the length of the platelet '1' obtained from Equation 9 and the axial stresses from Equation 7 are normalized by the applied stress and are plotted from the left end of platelet '1' to the right end of platelet '1'. The shear stresses for both, the dry and wet nacre, are plotted in Fig. 3 while the axial stresses are plotted in Fig. 4. It is seen that the shear and axial stress transfer for wet nacre (matrix with lower shear modulus, $G_3 = 1.4$ GPa) is more uniform than that of dry nacre (matrix with higher shear modulus, $G_3 = 4.6$ GPa). The shear stress concentration is higher at the ends of the unit cell for dry nacre while it is lower at the center of the unit cell as compared to wet nacre (Fig. 3). The axial stress concentration at the center of platelet '1' is similar for both the wet and dry nacre and is equal to twice the applied stress. From the assumptions, the axial stress at the end of the platelet '1' is zero in both cases. This is an approximation since the matrix at the ends of the platelets transfer axial loads. But the approximation is justified, as the extent of load transfer through the ends of the platelets is small.

The matrix at the ends of the platelets reaches the shear yield stress when the applied stresses are 69 and 49 MPa for wet and dry nacre respectively (Equation 13). The dry nacre yields at a lower applied stress in spite of having a higher shear yield stress as the shear stress concentration at the ends of the unit cell are higher for dry nacre when compared with wet nacre (Fig. 3). The experimentally measured strength values of wet and dry nacre are 130 and 167 MPa respectively [3]. Using these values and Equation 28, it is predicted that the composite fails when 'm' reaches 0.30 and 0.38 for the wet and dry nacre respectively (i.e. when the matrix is yielding over 59% and 77% of the platelet surface). The shear strain in the matrix at the ends of the unit cell obtained from Equation 22 and Equation 26 for these values of 'm' are found to be 8.5% and 7.1% respectively. We have assumed that when the shear strains in the matrix reach these values, the matrix fails. The remaining matrix cannot transfer the applied loads between the platelets and the composite fails. The ultimate shear strain values indicate that a large amount of plastic deformation takes place in the organic matrix. This is supported by the observation that the plastic deformation of the organic matrix resulting in relative motion of the aragonite platelets is responsible for the high toughness values obtained by nacre [2]. The large plastic deformation at the shear yield stress is due to averaging of the breaks in the loops or domains of the organic matrix fibers.

The stress-strain curves for wet and dry nacre are plotted in Fig. 5 until the experimentally attained ultimate stress values are reached. The point at which the matrix at the ends of the unit cell begins to yield is also shown. The axial stress at the center of platelet '1' at the failure point is 260 and 334 MPa for the wet and dry nacre respectively. Thus, the strength of an individual aragonite platelet is greater than 334 MPa. The presence of a small amount of matrix between the overlapped platelets with high ultimate shear strains enables the primarily aragonite solid to possess high values of toughness.

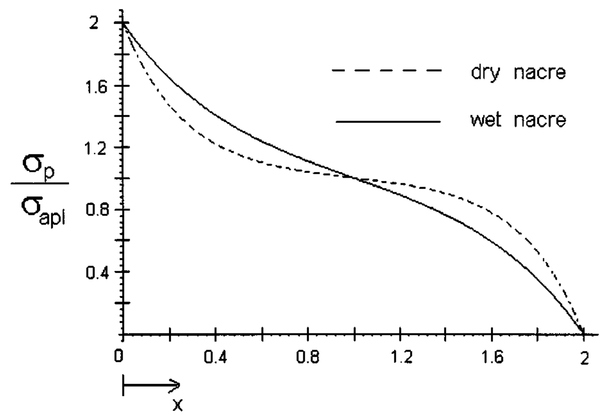

Figure 4 Axial stress transfer along platelet '1' from the right end to the left end of the unit cell.

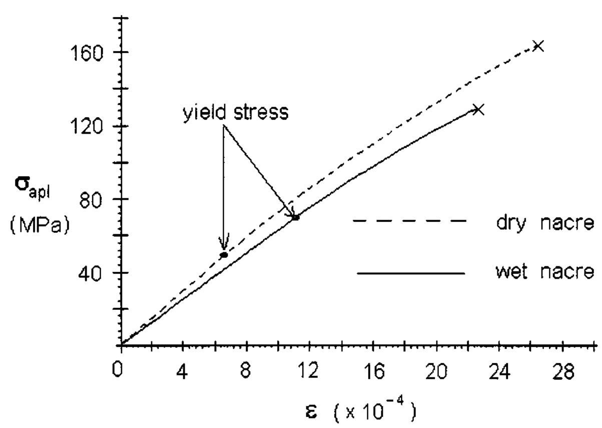

Figure 5 Tensile stress-strain behavior from the model.

4. Conclusion

In conclusion, a simple model has been developed to model the tensile behavior of *Pinctada* nacre. The developed theory agrees well with the experimentally obtained values. Composites having high values of toughness can be manufactured from platelets with small aspect ratios provided the matrix between the platelets could be engineered to have high shear strains. In the case of an organic matrix, high shear strains can be obtained through the presence of loops or domains that break at different strains.

Acknowledgements

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